EECE 340 Signals and Systems
American University of Beirut, Summer 2014–2015

July 28, 2015

Final Exam

Name:

ID:

Do not open this booklet until you are told to do so

- You have 2 hours to complete the exam.
- There are 3 problems on this exam. The problems are not in order of difficulty. Please read fully the problems before starting.
- A correct answer does not guarantee full credit. *Please indicate concisely but precisely your reasoning.* All answers need to be justified, unless you are specifically told otherwise,
- You may use FOUR double-sided "cheat-sheet" of size "A4" of *hand-written* notes.
- Calculators are allowed.
- Cell-phones are not allowed. Please turn them off and put them aside or give them to the staff to hold them for you until the end of the exam.

Problem F.1: (80 points)

The different parts of this problem are *INDEPENDENT*. Do any 4 of the 6 questions from (a) through (f). Do all for extra credit. The remaining questions (g) through (m) are all mandatory.

(a) Given a DT LTI system with input $x[n] = u[n]$ and unit sample response

$$h[n] = \left(\frac{1}{8}\right)^n u[n].$$

Find $y[0]$ and $\lim_{n\to+\infty} y[n]$.

(b) Given a CT LTI system with input $x(t) = e^{-(t+1)}u(t+1)$ and impulse response $h(t) = e^{-2t}u(t)$. Please determine the output $y(t)$.

(c) Given $x(t)$ and $h(t)$ are as plotted below (figure 1) ($h(t) = 1$ for $t > 2$ and $h(t) = 0$ for $t < -1$). Please determine $y(0)$ and $\lim_{t\to\infty} y(t)$.

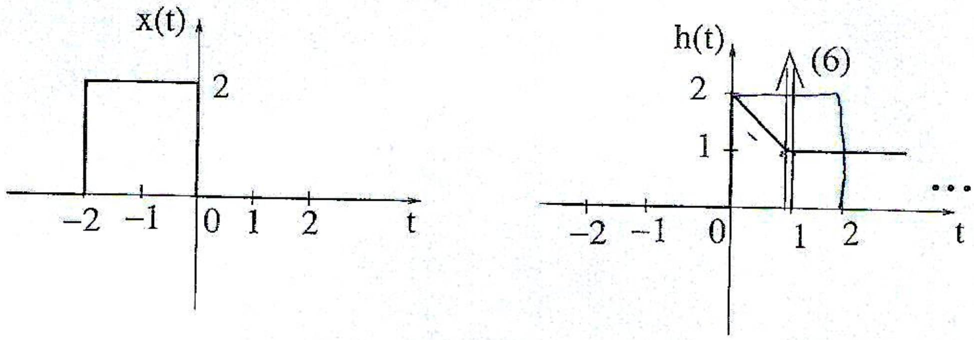


Figure 1: Part (c)

(d) Given the CT system shown in figure 2, where the input $x(t) = 2\frac{\sin t}{t}$, impulse train is $s(t) = \sum_{n=-\infty}^{\infty} \delta(t - \pi n)$, $x_s(t) = x(t)s(t)$,

and the impulse response $h(t) = \frac{\sin 3t}{t} - e^{-t}\cos(2t-3)$. Find the output $y(t)$

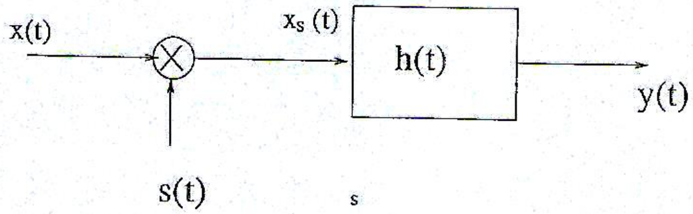


Figure 2: Part (d)

(e) You are given a Discrete time system where the Fourier transforms of the output $Y\left(e^{j2\pi f}\right)$ is related to that of the input by

$$Y\left(e^{j2\pi f}\right) = \int_{f-1/16}^{f+1/16} X\left(e^{j2\pi f}\right) df$$

Determine an expression relating $y[n]$ to $x[n]$.

(f) Suppose the CT input signal $x(t)$ is periodic and has the following Fourier Series:

$$x(t) = \sum_{n=-\infty}^{\infty} \left(\frac{|n|-3}{n^4+1}\right) e^{j\frac{2\pi}{T_0}nt} \qquad \text{where } T_o = 10$$

Also, suppose the system impulse response $h(t)$ has the Fourier transform $H(f)$ as given in figure 3 below ($H(f)$ is zero outside the region indicated). Please determine the output $y(t) = h(t) * x(t)$

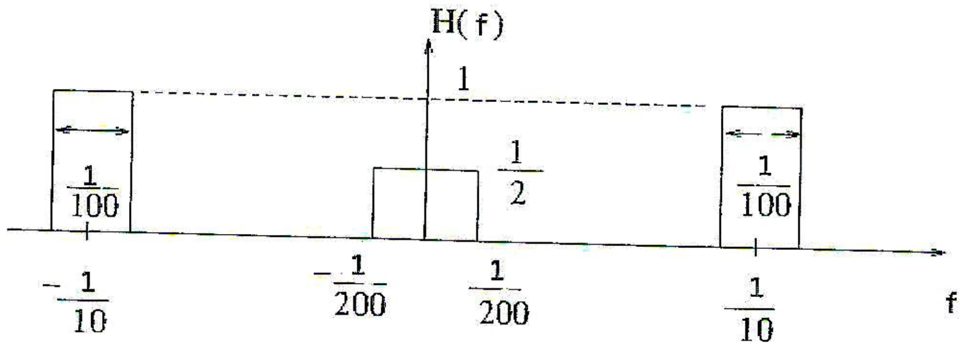


Figure 3: Part (f)

(g) Given a discrete time system whose input is $x[n]$ and output is $y[n]$. The Fourier transforms satisfy the following equation

$$Y\left(e^{j2\pi f}\right) = 3X\left(e^{j2\pi f}\right) + e^{-2j\pi f} X\left(e^{j2\pi f}\right) - \frac{dX\left(e^{j2\pi f}\right)}{df}$$

(a) Is this system linear? Justify clearly or show a counterexample.

(b) Is this system Time-invariant? justify clearly or show a counterexample.

(c) Determine the output $y[n]$ if the input is $x[n] = \delta[n]$.

(h) Consider the block diagram shown in figure 4.

(a) Find the system function of this CT LTI system.

(b) Find K_o and K_1 such that the overall system is stable.

(i) The following facts are known about a discrete time signal with a Z-transform $X(z)$.

(a) $x[n]$ is real and right sided.

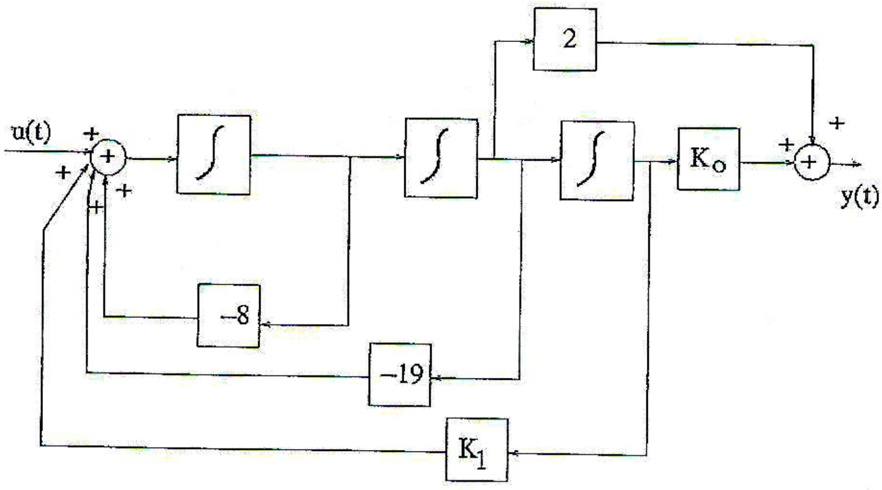


Figure 4: Block Diagram

(b) $X(z)$ has only two poles.
(c) $X(z)$ has two zeros at the origin.
(d) $X(z)$ has one pole at $z = \frac{1}{2}e^{j\pi/3}$.
(e) $X(1) = \frac{8}{3}$.

Determine $X(z)$ and its region of convergence.

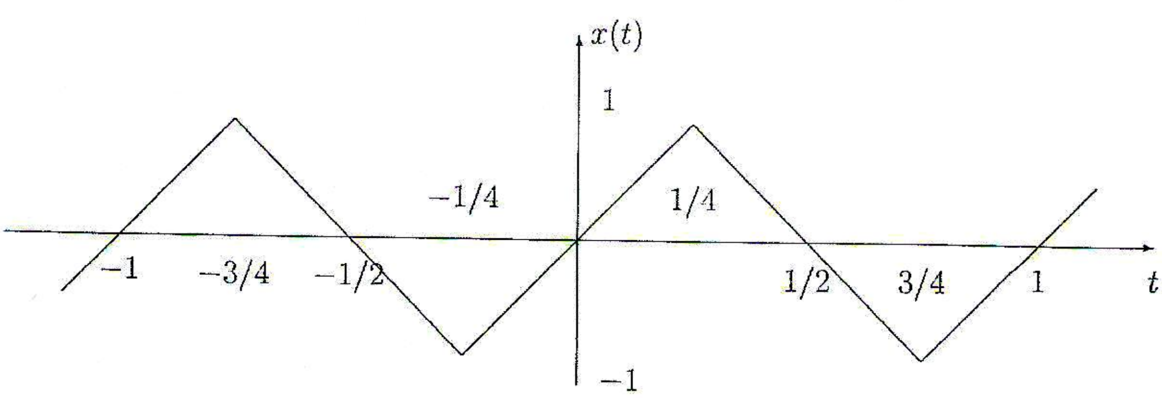
(j) A discrete time LTI system is described by the following linear difference equation

$$y[n] = \frac{1}{5}(x[n-1] + x[n-2] + x[n-4] - y[n-1] - y[n-2] - y[n-3] - y[n-4])$$

(a) Determine the output of this system if $x[n] = (-1)^n$ for all n.
(b) Determine the output of this system if

$$x[n] = \begin{cases} 2, & \text{n even;} \\ 1, & \text{n odd.} \end{cases}$$

(k) Consider the periodic triangular signal shown below:



(a) Determine the Fourier series coefficients a_k for $x(t)$.

(b) Consider a causal LTI system whose input-output relationship is by the following stable linear constant coefficient differential equation:

$$\frac{d^2y}{dt^2} + 4\pi\frac{dy}{dt} + 4\pi^2 y(t) = 4\pi^2 x(t),$$

where $x(t)$ is the input and $y(t)$ is the output of the system. **Let $x(t)$ be** the signal in part (a). Let b_k be the Fourier coefficients of the **corresponding** output $y(t)$. Find b_3 and b_3.

(l) Consider a system where its input $x(t)$ is first multiplied by

$$p(t) = \sum_{k \in \mathbb{Z}} \delta(t - kT),$$

and then low-pass filtered with an ideal filter with gain T:

$$H(f) = T\text{rect}(f/2B_c).$$

Suppose

$$x(t) = \frac{\sin(4\pi t)\sin(2\pi t)(-1)^t}{\pi^2 t^2}.$$

Determine B_c and a frequency B_o such that $y(t) = x(t)$ for any T such that $(1/2T) > B_o$.

(m) Consider the block diagram representation of a Discrete time causal LTI system shown in figure 5.

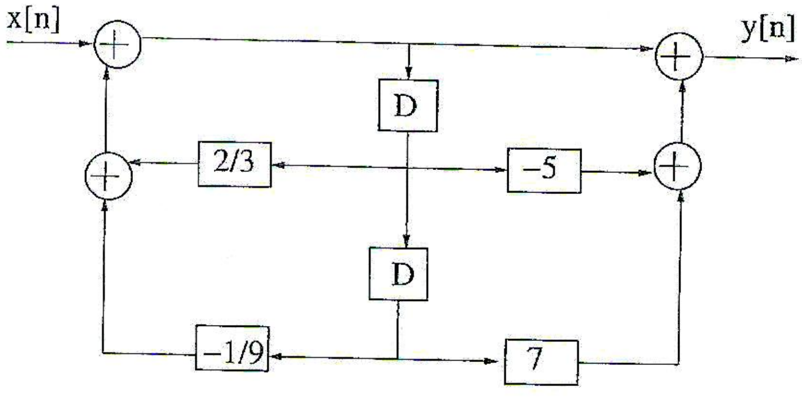


Figure 5: Block Diagram

(a) Find the Linear Difference equation describing the DT input/output relationship ($x[n]$ to $y[n]$).

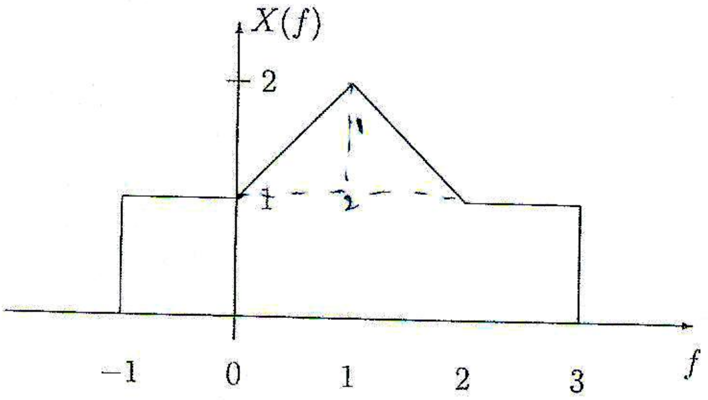
(b) Is this system stable? explain.

(c) Find the system output if $x[n] = (-1)^n \; -\infty < n < \infty$.

Problem F.2: (20 points)

Part I:

Consider the signal $x(t)$ whose Fourier transform is drawn below:



(a) Find the value of $x(0)$.

(b) Find the value of $\int_{-\infty}^{\infty} x(t)\,dt$.

(c) Find the value of $\int_{-\infty}^{\infty} |x(t)|^2\,dt$.

(d) Find the value of $\int_{-\infty}^{\infty} x(t)\,\frac{\sin(2\pi t)}{\pi t}\,e^{j2\pi t}\,dt$.

Part II: *Mildly dependent on Part I*

Consider the signal $y(t)$ whose Fourier transform is periodic with period equal to "5" and drawn below:

(e) Find the relationship between $y(t)$ and $x(t)$ defined in Part I ?

(f) Find the value of $y(0)$.

(g) Find the value of $\int_{-\infty}^{\infty} y(t)\, dt$. Is your answer sensible ? Explain.

(h) Find the value of $\int_{-\infty}^{\infty} |y(t)|^2\, dt$. Is your answer sensible ? Explain.

Part III: *Mildly dependent on Part I*

You decide to sample $x(t)$ (defined in Part I) every T seconds.

(i) What is the minimum rate at which you should sample in order to be able to recover $x(t)$? Explain.

(j) What is the DTFT of the sequence of samples $y[n] \hat{=} x(nT)$?

(k) Explain how you would reconstruct $x(t)$ from the sequence $y[n]$. Make sure to specify how the reconstruction is done in the time domain.

Problem F.3: (10 points)

Consider a system consisting of two systems cascaded in series: Sys A followed by Sys B.

Denoting the output of Sys A by $z(t)$, the input-output relation for Sys A is characterized by the following causal differential equation:

$$\frac{dz(t)}{dt} + 6z(t) = \frac{dx(t)}{dt} + 6x(t).$$

The impulse response of Sys B is given by:

$$h_B(t) = e^{-10t}u(t).$$

✓(a) What is the frequency response of the complete system? That is, find $Y(f)/X(f)$.

✓(b) What is the impulse response of the complete system?

✓(c) What is the difference equation that relates $x(t)$ and $y(t)$?

✓(d) Draw a block diagram of the complete system using minimal number of integrators.